

# Effect of temperature and flow nonuniformity on transient behaviour of crossflow heat exchanger

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## Abstract

The transient temperature response of a crossflow heat exchanger is carried out using finite difference method accounting for the effect of temperature and flow nonuniformity at different input conditions. Beta flow maldistribution model has been introduced for the flow nonuniformity. The responses are found dependent on the relative position of the individual temperature streams and the position of the fluid moving device for the temperature and flow nonuniformity, respectively. Combined effect of temperature and flow nonuniformity has also been analysed and compared with the other cases.

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## 1. Introduction

Transient response of heat exchangers needs to be known for designing the control strategy of different HVAC (heating ventilation and air conditioning) systems, cryogenic and chemical process plants. Problems such as start-up, shut-down, failure and accidents have motivated investigations of transient thermal response in crossflow heat exchangers. The situation is more serious when nonuniformity is present in the temperature and/or flow at the entry. The temperature and fluid flow distribution through the heat exchangers are usually nonuniform under the actual operating conditions. So, the transient response with temperature and flow nonuniformity will help the designer to rely on a solution, for the time dependent temperature problems, very useful in thermal and stress analyses.

For solving the transient equations different methods have been adopted. The solution of basic governing equations was carried out numerically by Myers et al. [1], Yamashita et al. [2] and Kou and Yuan [3]. Myers et al. [4] used an approximate integral approach to solve the transient equations for large wall capacitance. Romie [5,6] and Spiga and Spiga [7–9] used the Laplace transformation of the governing equations for gas-to-gas crossflow heat exchangers with finite and large core capacitance. Chen and Chen [10,11] also used the Laplace transform method but they have used numerical inversion technique for solving the transformed temperatures. The case of flow nonuniformity was first investigated by Chiou [12] for the steady state condition. Similarly the case of nonuniform inlet temperature was taken up by Kou and Yuan [13] for finding out the effects of longitudinal conduction again at steady state condition. Ranganayakulu et al. [14] and Ranganayakulu and Seetharamu [15] have shown the effect of flow nonuniformity with and without core longitudinal conduction on the thermal performance of crossflow plate-fin heat exchangers using finite element method. Ranganayakulu and Seetharamu [16] have further investigated the

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## Nomenclature

|            |   |                      |  |
|------------|---|----------------------|--|
| $A$        | area of heat transfer, $\text{m}^2$                                     | $Y$                  | $= \left(\frac{hA}{mc}\right)_b \frac{y}{L_b}$ , dimensionless length  |
| $A_c$      | area of cross-section, $\text{m}^2$                                     | <i>Greek symbols</i> |  |
| $C$        | specific heat of the wall material, $\text{J/kg K}$                     | $\alpha$             | flow maldistribution factor ( $=m'/m$ )  |
| $c, c_p$   | isobaric specific heat of fluid, $\text{J/kg K}$                        | $\beta$              | constant (0.8 for the present calculation)   |
| $D$        | axial dispersive diffusion coefficient, $\text{W/m K}$                  | $\beta(p, q)$        | Beta function as defined in Eq. (25)   |
| $E$        | capacity rate ratio $= \frac{(mc)_b}{(mc)_a}$                           | $\epsilon$           | effectiveness  |
| $G$        | mass flux velocity, $\text{kg/m}^2 \text{s}$                            | $\eta_0$             | efficiency   |
| $h$        | heat transfer coefficient, $\text{W/m}^2 \text{K}$                      | $\lambda$            | longitudinal heat conduction parameter,<br>$\lambda_{\cdot a} = \frac{k\delta \cdot L_b}{L_a(mc)_a}$ , $\lambda_{\cdot b} = \frac{k\delta \cdot L_a}{L_b(mc)_b}$ |
| $k$        | thermal conductivity of the separating sheet, $\text{W/m K}$            | $\mu$                | dynamic viscosity, $\text{N s/m}^2$  |
| $L$        | heat exchanger length, $\text{m}$                                       | $\rho$               | density, $\text{kg/m}^3$   |
| $m$        | mass flow rate of fluid, $\text{kg/s}$                                  | $\tau$               | time, $\text{s}$   |
| $M$        | mass of the separating sheet, $\text{kg}$                               | $\phi(\cdot)$        | perturbation in hot fluid inlet temperature  |
| $N_a, N_b$ | as defined in Eqs. (5)–(8)  | $\theta$             | $= \frac{(hA)_{ax}}{MC}$ , dimensionless time  |
| NTU        | number of transfer units  | <i>Subscripts</i>    |  |
| $Pe$       | axial dispersive Peclet number $= \frac{(mc)L}{A_c D}$                  | $a, b$               | side $a$ and $b$   |
| $R$        | conductance ratio $= \frac{(hA)_b}{(hA)_a}$                             | $c, h$               | cold and hot   |
| $Re$       | Reynolds number   | $w$                  | wall   |
| $t$        | temperature, $^\circ\text{C}$   | $in$                 | inlet value  |
| $T$        | $= \frac{t - t_{b,in}}{t_{REF} - t_{b,in}}$ , dimensionless temperature | $ex$                 | exit value   |
| $\bar{t}$  | mean temperature  | $min$                | minimum  |
| $\bar{T}$  | mean dimensionless temperature  | <i>Superscript</i>   |  |
| $U$        | overall heat transfer coefficient, $\text{W/m}^2 \text{K}$              | '                    | final value  |
| $u, v$     | velocity in $x$ - and $y$ -direction                                    |                      |  |
| $V$        | capacitance ratio $= \frac{LA_c \rho \cdot c}{MC}$                      |                      |  |
| $X$        | $= \left(\frac{hA}{mc}\right)_a \frac{x}{L_a}$ , dimensionless length   |                      |  |
| $x, y$     | direction, lengths from the entry                                       |                      |  |

combined effect of longitudinal conduction, flow and temperature nonuniformity on steady state performance of crossflow plate–fin heat exchangers. Roetzel and Xuan [17] analysed the dynamic behaviour of crossflow heat exchangers to calculate the outlet temperature response to arbitrary inlet temperature and flow rate disturbances. Solution methodologies by Laplace transform as well as finite difference scheme have been discussed. Effects of flow maldistribution and wall heat conduction resistance have also been discussed and analysed. The effect of different flow maldistribution models on the thermal performance of three-fluid crossflow heat exchanger has been studied by Yuan [18]. Further, transient response of plate heat exchangers considering the effect of flow maldistribution has been analysed by Srihari et al. [19] but to the best of authors' knowledge the effect of temperature or flow nonuniformity on the transient behaviour of crossflow heat exchangers has not been analysed so far.

The present work analyses the direct transfer, single pass crossflow heat exchanger with both fluids unmixed having finite capacitance wall separating the two fluid streams. Individual as well as combined effect of one-dimensional inlet temperature and flow nonuniformity has been carried out numerically using finite difference method to get the

transient response for step, ramp and exponential inputs given to the hot fluid inlet temperature. The combined effect of two-dimensional longitudinal conduction in wall and fluid axial dispersion has also been considered for solution.

## 2. Mathematical formulation

A direct transfer, two fluid, crossflow, multilayer plate–fin heat exchanger is shown schematically in Fig. 1a. Following assumptions are made for the mathematical analysis:

1. Both fluids are single phase, unmixed and do not contain any volumetric source of heat generation.
2. The exchanger shell or shroud is adiabatic and the effects of the asymmetry in the top and bottom layers are neglected. Therefore, the heat exchanger may be assumed to comprise of a number of symmetric sections as shown in Fig. 1b in schematic form.
3. The thermo-physical properties of both fluids and walls are constant and uniform.
4. The primary and secondary areas of the separating plate have been lumped together, so that the variation of wall temperature is also two-dimensional.

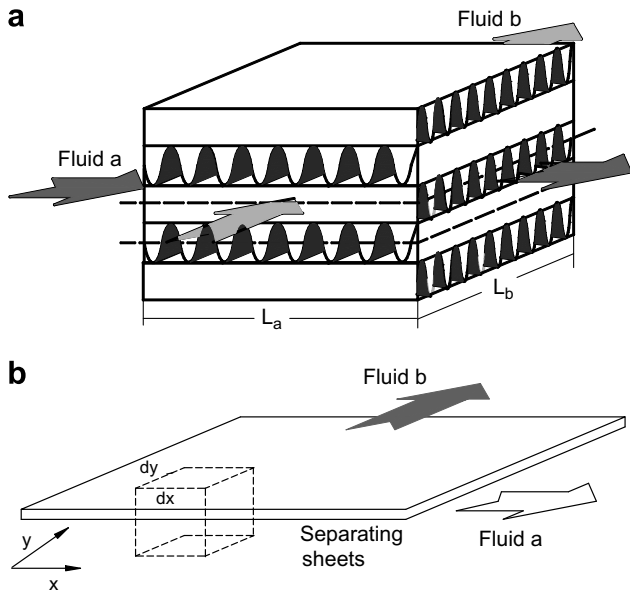


Fig. 1. Crossflow heat exchanger (a) schematic representation, and (b) symmetric module considered for analysis.

5. Heat transfer area per unit base area and surface configurations are constant.
6. Variation of temperature in the fluid streams in a direction normal to the separating plate is neglected.
7. In case of temperature nonuniformity, the hot fluid inlet is assumed to consist of two streams of the same uniform velocity but at different temperature levels.
8. In case of flow nonuniformity, the convection heat transfer coefficient between fluids and their respective heat transfer surfaces is directly proportional to the mass flux velocity of the fluid raised to the power  $\beta$ , ( $h \propto G^\beta$ ).
9. Thermal and dispersive disturbances propagate with infinite velocity.

Conservation of energy for wall and two fluid streams considering longitudinal conduction in separating sheet and the axial dispersion in fluids can be expressed in non-dimensional form as given below:

$$\frac{\partial T_w}{\partial \theta} = T_a + R \cdot T_b - (1 + R) \cdot T_w + \lambda_a \cdot N_a \frac{\partial^2 T_w}{\partial X^2} + \lambda_b \cdot N_b \cdot R \cdot \frac{\partial^2 T_w}{\partial Y^2}, \quad (1)$$

$$V_a \cdot \frac{\partial T_a}{\partial \theta} = T_w - T_a - \frac{\partial T_a}{\partial X} + \frac{N_a}{Pe_a} \frac{\partial^2 T_a}{\partial X^2}, \quad (2)$$

$$\frac{V_b}{R} \frac{\partial T_b}{\partial \theta} = T_w - T_b - \frac{\partial T_b}{\partial Y} + \frac{N_b}{Pe_b} \frac{\partial^2 T_b}{\partial Y^2}, \quad (3)$$

where non-dimensional terms are defined as

$$X = \left(\frac{hA}{mc}\right)_a \frac{x}{L_a} = N_a \frac{x}{L_a}, \quad Y = \left(\frac{hA}{mc}\right)_b \frac{y}{L_b} = N_b \frac{y}{L_b},$$

where

$$N = \frac{hA}{mc}, \quad \theta = \frac{(hA)_a \tau}{MC}, \quad T = \frac{t - t_{b,in}}{t_{REF} - t_{b,in}},$$

$$\text{Conductance Ratio, } R = \frac{(hA)_b}{(hA)_a},$$

$$\text{Capacitance Ratio, } V = \frac{LA_c \rho c}{MC},$$

Longitudinal Heat Conduction Parameter,

$$\lambda_{.a} = \frac{k\delta \cdot L_b}{L_a(mc)_a}, \quad \lambda_{.b} = \frac{k\delta \cdot L_a}{L_b(mc)_b},$$

$$\text{Axial Dispersive Peclet number, } Pe = \frac{(mc)L}{A_c \cdot D}.$$

NTU is defined as

$$\frac{1}{NTU} = C_{\min} \left[ \frac{1}{(hA)_a} + \frac{1}{(hA)_b} \right]. \quad (4)$$

Further  $N_a$  and  $N_b$  can be expressed as a function of non-dimensional heat exchanger parameters namely number of transfer units (NTU), conductance ratio ( $R$ ) and capacity rate ratio

$$\left[ E = \frac{(mc)_b}{(mc)_a} \right].$$

For  $C_a = C_{\min}$

$$N_a = NTU \left( 1 + \frac{1}{R} \right), \quad (5)$$

$$N_b = \frac{NTU}{E} (R + 1) \quad (6)$$

for  $C_b = C_{\min}$

$$N_a = NTU \cdot E \left( 1 + \frac{1}{R} \right), \quad (7)$$

$$N_b = NTU(1 + R). \quad (8)$$

Eqs. (1)–(3) are subjected to following initial and boundary conditions:

$$T_a(X, Y, 0) = T_b(X, Y, 0) = T_w(X, Y, 0) = 0, \quad (9)$$

$$\frac{\partial T_a(X, Y, \theta)}{\partial X} \Big|_{X=N_a} = 0, \quad (10)$$

$$\frac{\partial T_b(X, Y, \theta)}{\partial Y} \Big|_{Y=N_b} = 0, \quad (11)$$

$$\frac{\partial T_w(X, Y, \theta)}{\partial X} \Big|_{X=0} = \frac{\partial T_w(X, Y, \theta)}{\partial X} \Big|_{X=N_a} = \frac{\partial T_w(X, Y, \theta)}{\partial Y} \Big|_{Y=0} = \frac{\partial T_w(X, Y, \theta)}{\partial Y} \Big|_{Y=N_b} = 0, \quad (12)$$

$$T_a(0, Y, \theta) = \phi(\theta), \quad (13)$$

$$T_b(X, 0, \theta) = 0. \quad (14)$$

Solution may be obtained for any arbitrarily specified temperature function  $\phi(\theta)$ . However, dynamic response of heat exchanger is generally looked for step, ramp and exponential variation of temperature. Such variation may occur during operations or they may be especially created

for the purpose of transient testing of heat exchangers. Though a ramp or an exponential function gives a continuous increase in temperature, such an increase for a prolonged duration is not feasible in reality. For instance the initial temperature rise may have the ramp or the exponential nature in both designed and unforeseen transients, but the maximum value of temperature rise will generally not be unlimited. In the present study a limit of maximum temperature has been considered [20]. Additionally sinusoidal input function has also been tried for the temperature responses. Accordingly the functional form of  $\phi(\theta)$  is expressed as follows:

$$\phi(\theta) = \begin{cases} 1 & \text{for step input,} \\ \begin{cases} \alpha\theta, & \theta \leq 1 \\ 1, & \theta > 1 \end{cases} & \text{for ramp input,} \\ 1 - e^{-\alpha\theta} & \text{for exponential input,} \\ \sin(\alpha\theta) & \text{for sinusoidal input,} \end{cases} \quad (15)$$

where  $\alpha$  is assumed to be unity in the present analysis.

### 3. Method of solution

The conservation equations are discretised using the implicit finite difference technique [21]. Forward difference scheme is used for time derivatives, while upwind scheme and central difference scheme are used for the first and second order space derivatives, respectively. The difference equations along with the boundary conditions are solved using Gauss Seidal iterative technique. The convergence of the solution has been checked by varying the number of space grids and size of the time steps. The solution gives the two-dimensional temperature distribution for both the fluids as well as for the separator plate. Additionally one may calculate the mean exit temperatures as follows:

$$\bar{T}_{a,ex} = \frac{\int_0^{N_a} T_{a,ex} \cdot u \, dy}{\int_0^{N_a} u \, dy} \quad \text{and} \quad \bar{T}_{b,ex} = \frac{\int_0^{N_b} T_{b,ex} \cdot v \, dx}{\int_0^{N_b} v \, dx} \quad (16)$$

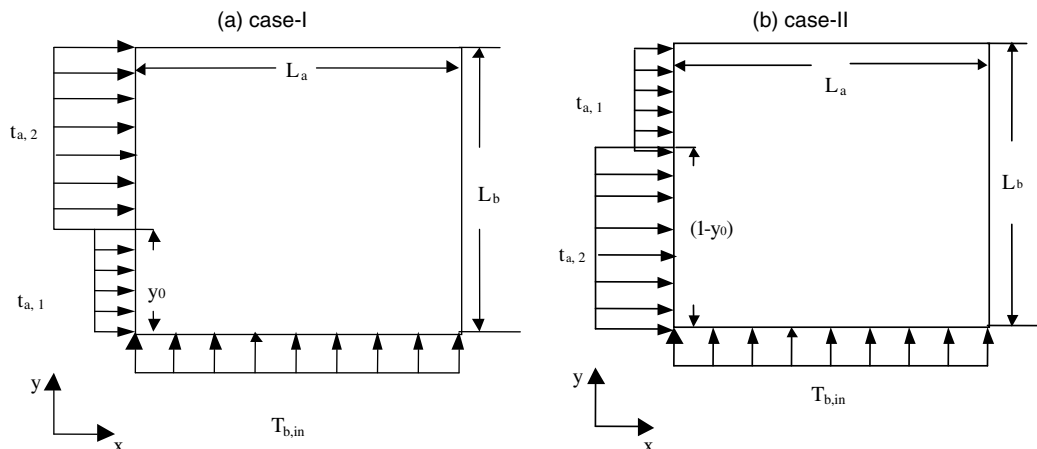


Fig. 2. Schematic diagrams showing nonuniformity in temperature by changing relative positions of  $t_{a,1}$  and  $t_{a,2}$  (a) case I, and (b) case II.

To check the validity of the numerical scheme, the results of the present investigation have been compared with available analytical results. For balanced gas-to-gas crossflow heat exchangers, Spiga and Spiga [7] determined the variation of exit temperature in the absence of core longitudinal conduction and fluid axial dispersion for a conductance ratio of 1 using Laplace transform. An excellent agreement [20] has been obtained between the results of present investigation and those obtained by Spiga and Spiga [7] for step, ramp and exponential inputs.

### 4. Results and discussion

Performance of the heat exchanger was studied over a wide range of parameters as well as for sufficient time duration so that steady state conditions are obtained for each individual excitation. Some of the salient results are discussed below.

#### 4.1. Temperature nonuniformity

The hot and cold fluids enter their respective layers of the core by the header and flow distributors. In general the inlet temperatures of both the fluids are assumed to be uniform. Various researchers have considered the thermal performance of crossflow heat exchanger with uniform inlet temperatures. Many a times the fluid entering to the core have more than one stream and the complete mixing does not take place before entering the heat exchanger. The inlet temperature becomes nonuniform when two fluid currents at different temperature enter into the heat exchanger core without complete mixing. The steady state thermal performance is affected due to nonuniformity of temperature and is presented by Kou and Yuan [13]. At the same time its effect cannot be ignored in transient state also. To examine the effect of inlet temperature distribution on the transient performance of the heat exchanger three different cases have been considered. In all the three cases, the mean inlet temperature of the hot fluid is the same.

However, in two cases the temperature distributions are nonuniform as shown in Fig. 2. In the third case (case III) the temperature distribution is uniform.

For both cases I and II a stepped temperature distribution specified by two temperature values  $t_{a,1}$  and  $t_{a,2}$  and a known dimension  $y_0$  are taken. The dimensionless temperature is defined as

$$T = \frac{t - t_{b,in}}{t_{a,in} - t_{b,in}}, \quad (17)$$

where

$$\overline{t_{a,in}} = \frac{t_{a,1} \cdot y_0 + t_{a,2} \cdot (L_b - y_0)}{L_b} \quad (18)$$

Therefore, in case III, a uniform dimensionless inlet temperature is given by

$$\overline{T_{a,in}} = T_{a,1} \cdot Y_0 + T_{a,2} \cdot (1 - Y_0). \quad (19)$$

The temperature distribution of the cold fluid for all the above cases

$$T_b(X, 0) = T_{b,in} = 0. \quad (20)$$

To study the transient performance of the heat exchanger following input condition for the hot fluid inlet temperature is considered:

$$\overline{T_{a,in}} = T_{a,1} \cdot Y_0 + T_{a,2} \cdot (1 - Y_0) = \phi(\theta), \quad (21)$$

where  $\phi(\theta)$  is a specified function of temperature with respect to time. To get the temperature distribution at the inlet one needs to supply the values of  $T_{a,1}$  and  $Y_0$ . Four different temporal forms of  $\phi(\theta)$  namely step, ramp, exponential and sinusoidal variations are considered in the present work.

To check the validity of the numerical scheme for temperature nonuniformity, the results of the present investigation have been compared with available steady state results. For  $Y_0 = 0.5$  and  $T_{a,1} = 0.6$  ( $T_{a,2} = 1.4$ ), the solution of Eqs. (1)–(3) for the two relative positions of  $T_{a,1}$  and  $T_{a,2}$  shows a good match with the steady state solution given by Kou and Yuan [13] as shown in Fig. 3.

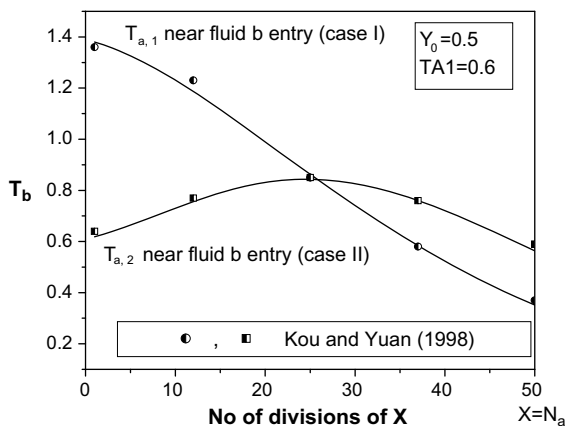


Fig. 3. Effect of temperature nonuniformity compared with the steady state solution of Kou and Yuan [13].

For  $Y_0 = 0.2$  and  $T_{a,1} = 0.1$  the solution for the transient condition at different inputs are shown in Fig. 4. For all the four types of excitations it may be observed that the mean exit temperature of the hot fluid is influenced only marginally by the nonuniformity at the entry. On the other hand the effect of nonuniformity is pronounced in case of exit temperature of cold fluid. In all the three cases of step, ramp and exponential excitation the mean exit temperature of the cold stream is highest in case I. Case II gives the lowest mean exit temperature, while case III falls in between. It may be noted that due to nonuniform distribution of temperature at the hot stream entry, the mean exit temperature of the cold stream receives more heat at the exit amongst all the arrangements in case I. This clearly shows that the cold fluid exit temperature is decided not by the mean inlet temperature of the heat exchanger and the process and geometrical parameters only but also by the temperature distribution at the hot fluid inlet. The effect of nonuniformity is visible also in the case of sinusoidal excitation. The effect on cold fluid exit temperature is relatively more significant with the maximum amplitude of exit temperature obtained for case I and minimum for case II.

#### 4.2. Flow nonuniformity

The fluid flow distribution over the heat exchanger core is usually nonuniform under actual operating conditions. The reasons for flow nonuniformity are the improper exchanger entrance configuration and imperfect flow passage caused by various problems in design, manufacturing or fouling. It can be avoided up to some extent by adopting a suitable design of the header. But many a times uniform flow prior to entry section cannot be ensured due to space or some other constraints. In those cases, the nonuniformity is well governed by entry of the fluid into the core through the fluid moving device and the configuration of the connecting conduits. The flow nonuniformity can be on one side or on both the sides. Different models of flow nonuniformity have been proposed [12,14,15] for studying the thermal performance of the crossflow heat exchanger at steady state condition. The present work is an extension for the transient condition with different types of disturbances provided to hot fluid inlet temperature.

In the present study it is assumed that the cold fluid moving in  $y$ -direction is nonuniformly distributed, and the other fluid is uniformly distributed. It is further assumed that the analysis is assumed to be restricted to the cases when the flow regime in the exchanger is predominantly fully developed turbulent flow. Thus the convection heat transfer coefficient  $h$  is considered to be proportional to  $G^\beta$  ( $\beta = 0.8$ ). However, the analysis and the equations presented can be applied to any flow pattern if appropriate values for  $\beta$  are used. The value of one-dimensional  $\alpha$  shown in Fig. 5 is from the wind tunnel experimentation given by Chiou [12] for the case when the flow inlet manifold is at the centre of the core.

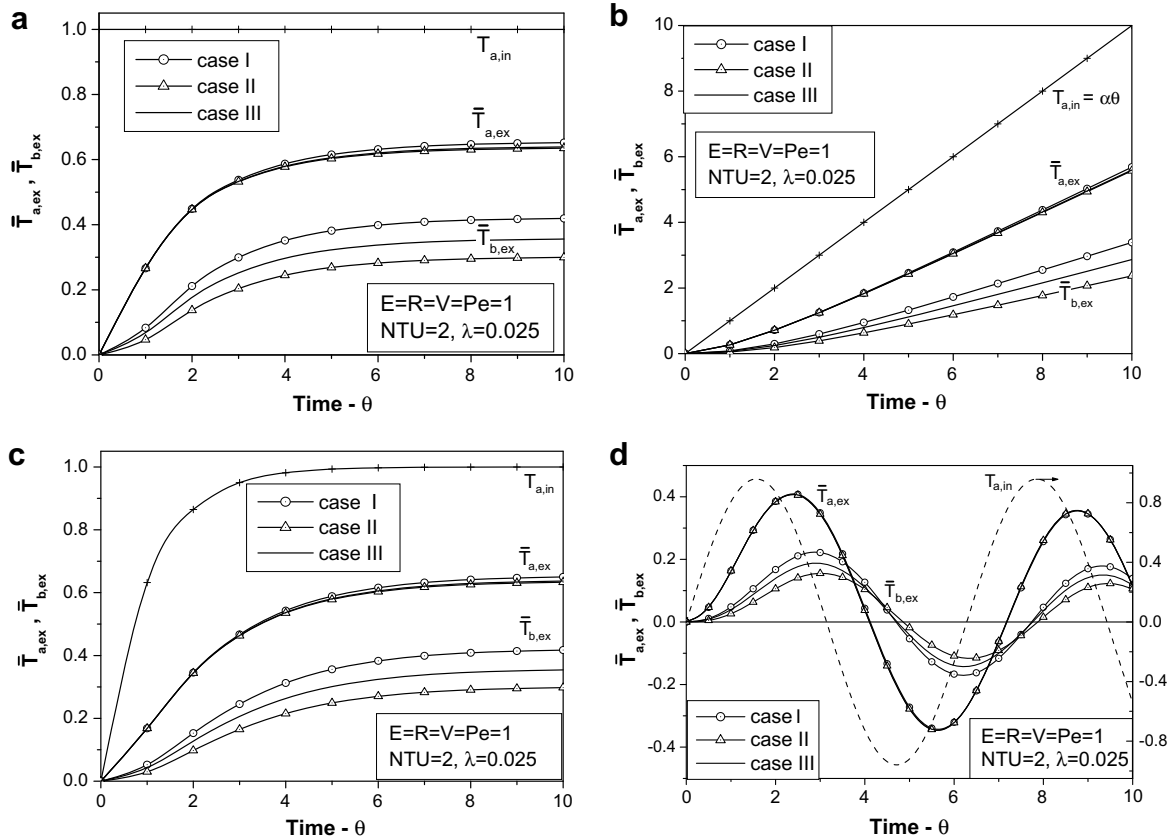


Fig. 4. Effect of temperature nonuniformity on mean exit temperature of hot and cold fluids for (a) step, (b) ramp, (c) exponential, and (d) sinusoidal inputs given to the hot fluid.

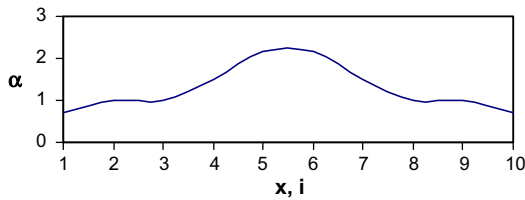


Fig. 5. Flow distribution model [12].

For the cold fluid *b* (in which the nonuniformity is taking place) mass flow rate,  $m'_b = \alpha \cdot m_b$ , and heat transfer coefficient  $h'_b = \alpha^\beta \cdot h_b$ , where  $\alpha$  is the maldistribution factor ( $= m'_b/m_b$ ). The new values of mass flow rate ( $m'_b$ ) and heat transfer coefficient ( $h'_b$ ) when substituted to basic governing equations of the energy conservation in the wall and the two fluids give following equations in the dimensionless form:

$$\frac{\partial T_w}{\partial \theta} = T_a + R \cdot T_b \cdot \alpha^\beta - (1 + R \cdot \alpha^\beta) \cdot T_w + \lambda_a \cdot N_a \frac{\partial^2 T_w}{\partial X^2} + \lambda_b \cdot N_b \cdot R \cdot \alpha^{2(\beta-1)} \cdot \frac{\partial^2 T_w}{\partial Y^2}, \quad (22)$$

$$V_a \cdot \frac{\partial T_a}{\partial \theta} = T_w - T_a - \frac{\partial T_a}{\partial X} + \frac{N_a}{Pe_a} \frac{\partial^2 T_a}{\partial X^2}, \quad (23)$$

$$\frac{V_b}{R} \frac{\partial T_b}{\partial \theta} = \alpha^\beta \cdot (T_w - T_b) - \alpha \frac{\partial T_b}{\partial Y} + \frac{N_b}{Pe_b} \alpha^{2(\beta-1)} \frac{\partial^2 T_b}{\partial Y^2}. \quad (24)$$

The solution of the above equations, using the flow maldistribution model, shown in Fig. 5, are depicted in Fig. 6 for the same initial and boundary conditions used in Eqs. (9)–(14).

The variation of mean exit temperatures show that the effect of flow maldistribution is predominant on cold fluid as nonuniformity is assumed only on cold side. The decrease in mean exit temperature of cold fluid and a slight increase in hot fluid mean exit temperature shows the deterioration in the performance and in turn reduction in heat transfer between the two fluids. The responses are similar for step, ramp and exponential inputs due to the specific nature of the function  $\phi(\theta)$  defined in Eq. (15).

#### 4.2.1. Beta distribution model for flow maldistribution

The flow distribution considered by Chiou [12] was based on experimental observation. Therefore, it is suitable for a particular flow geometry and test condition and lacks generality. On the other hand, researchers [14–16,18,19] have considered different theoretical models for flow maldistribution. One-dimensional Beta distribution model of the first kind could be a good alternative because of its single mode, finite limits and the tendency to be skewed positively or negatively [22]. The Beta function,  $\beta(p, q)$  of the parameters  $p$  and  $q$ , is defined as

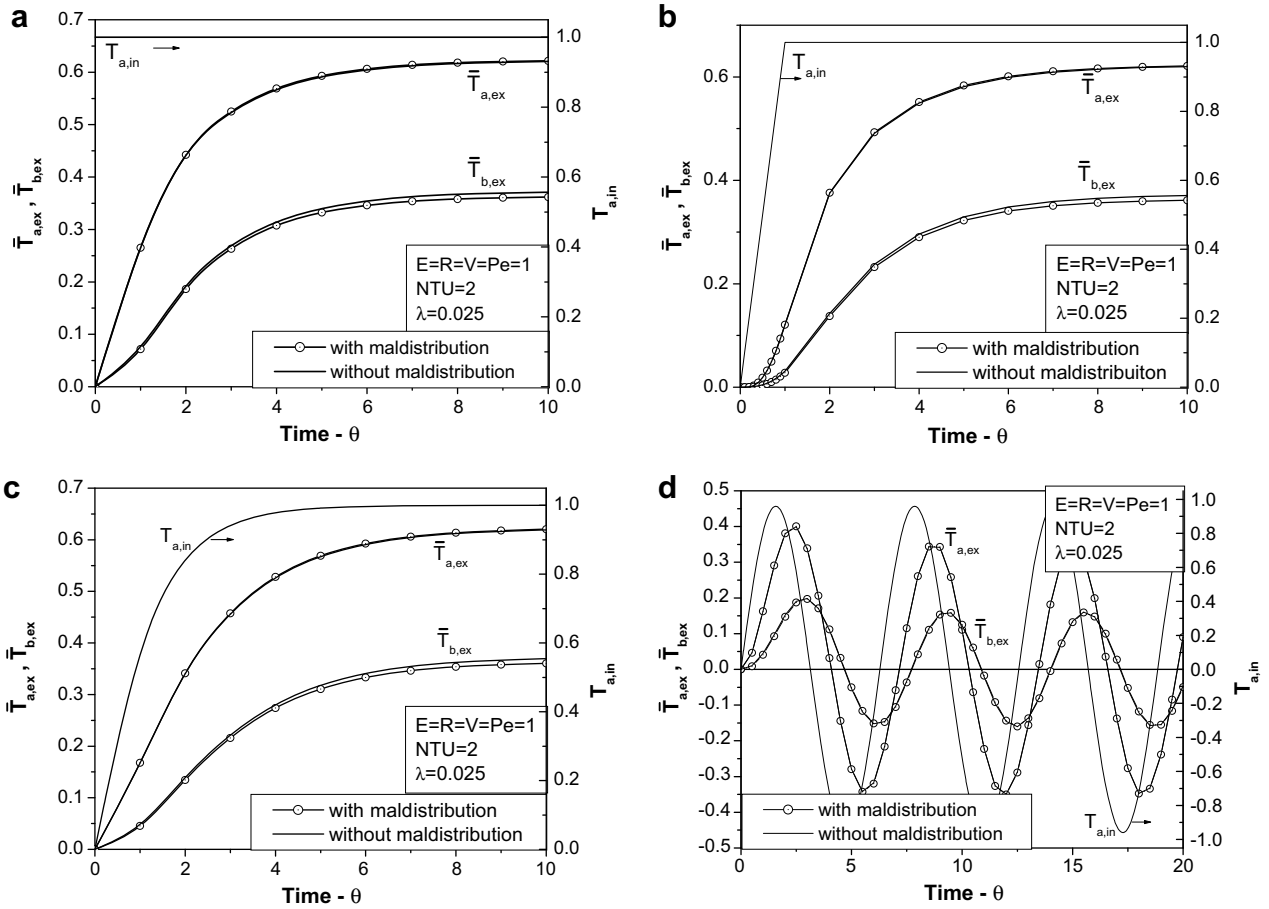


Fig. 6. Effect of Chiou's flow maldistribution model (Fig. 5) on hot and cold fluid mean exit temperatures for (a) step, (b) ramp, (c) exponential, and (d) sinusoidal inputs.

$$\beta(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx. \tag{25}$$

$$f(x) = \frac{1}{\beta(p, q)} [x^{p-1} (1-x)^{q-1}], \quad 0 \leq x \leq 1 \tag{26}$$

Fig. 7 shows the probability density function for a few selected values of  $p$  and  $q$ . The mass flow rate of the fluid moving in  $y$ -direction is assumed to follow the Beta distribution of first kind as given below:

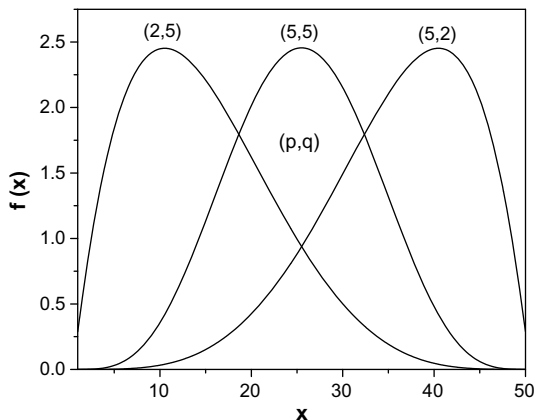


Fig. 7. Probability density function for Beta distribution of first kind with some pairs of  $p, q$ .

Depending upon the combination of values of  $(p, q)$ , the peak of the flow distribution curve shifts towards left (2, 5), right (5, 2) or remains at centre (5, 5). In practical situations these conditions may be obtained by a change in the position of fluid moving device or by a bend occurring before the entry to the heat exchanger.

In practical situations, especially with offset-strip fin surfaces, the effect of flow maldistribution will neither be only at the entry nor it will travel fully up to the exit section, but it travels up to a certain length. In the absence of the exact length up to which the effect should be considered, for the results shown in Fig. 6 and for other results to follow, the flow nonuniformity is assumed to travel throughout up to the heat exchanger exit section.

As an example, the effect of flow maldistribution on step response of hot and cold fluids are shown in Fig. 8 comparing the case when maldistribution effect is only at the entry with the case when it travels up to the exit section. It is clear that considering the effect only at the entry does not show any change in the responses. This suggests for considering the effect up to the exit section in absence of the

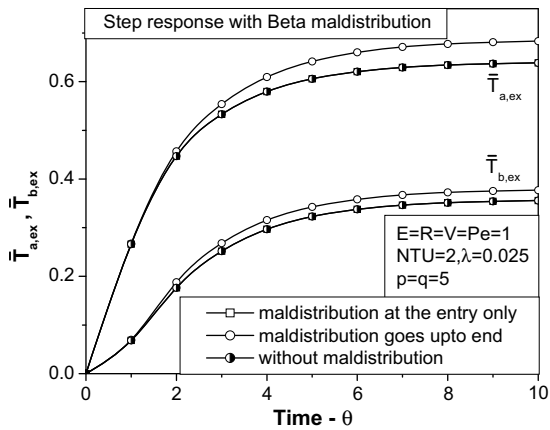


Fig. 8. Comparison of the step response when the flow maldistribution is considered only at the entry to that when it travels up to the exit of the heat exchanger.

knowledge of actual length of travel. The actual response will lie in between these two extreme responses shown in figure. Further, Fig. 9a–d shows the effect of Beta flow maldistribution on the temperature responses with different

input conditions for different combinations of  $(p, q)$  at  $E = R = V = Pe = 1$ ,  $NTU = 2$  and  $\lambda = 0.025$ .

From the Beta flow distribution model it is clear that for the curve showing  $(p, q)$  combination (5, 5), the position of the fluid moving device is at the centre, (2, 5) shows that the device is shifted towards left i.e. towards hot fluid entry, and (5, 2) shows the device away from the hot fluid entry. The difference between hot and cold fluid mean exit temperatures is almost same for all the three positions, but if cold fluid mean exit temperature is the parameter of interest, (2, 5) is better and (5, 2) is worse. It means that as the fluid moving device is moved away from the hot fluid entry side the performance is worse in terms of cold fluid mean exit temperature.

4.3. Combined temperature and flow nonuniformity

So far, nonuniformity in either temperature or flow has been considered at a time. Now, the present scheme includes the combined effect of temperature and flow nonuniformity in a crossflow heat exchanger. As shown in schematic diagram in Fig. 10, temperature nonuniformity

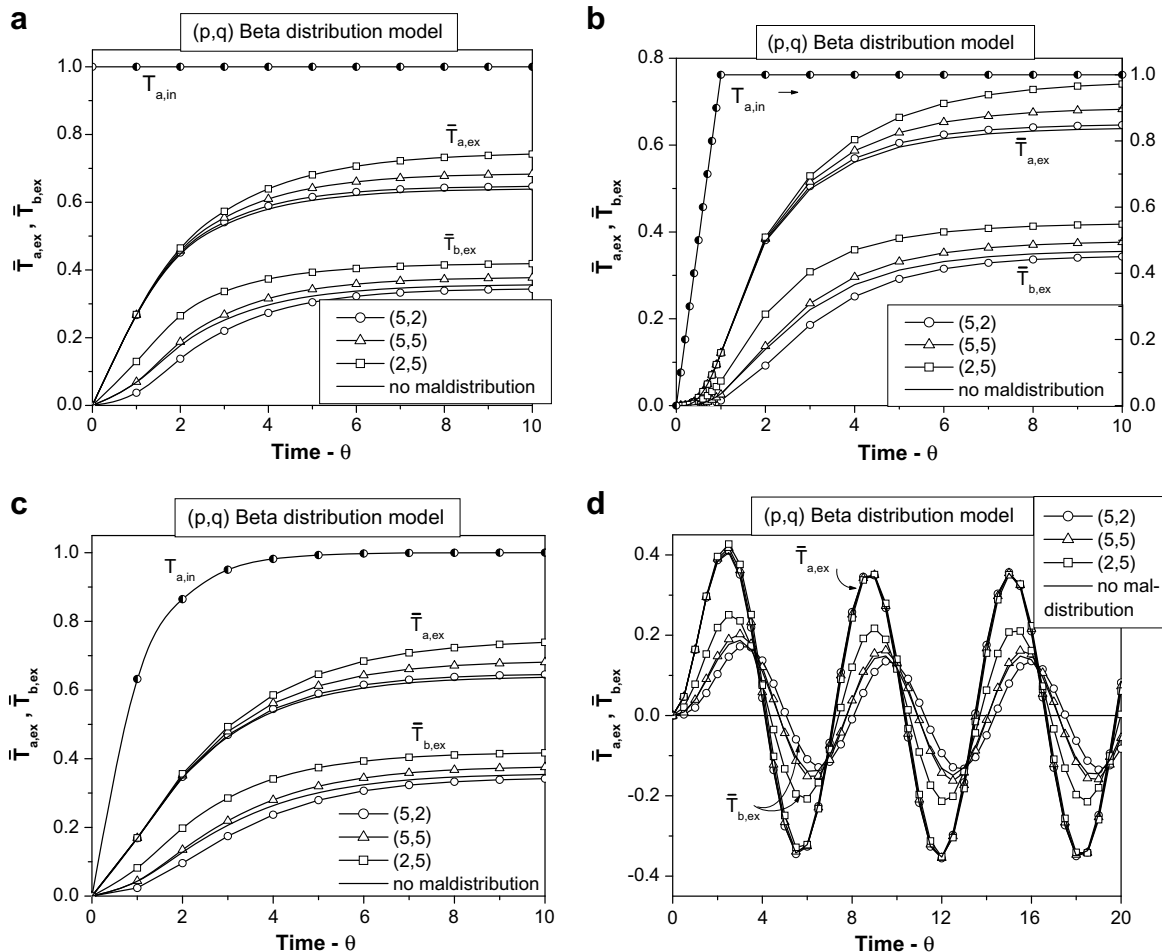


Fig. 9. Effect of Beta flow maldistribution model on hot and cold fluid mean exit temperatures for (a) step, (b) ramp, (c) exponential, and (d) sinusoidal inputs ( $E = R = V = Pe = 1$ ,  $NTU = 2$ ,  $\lambda = 0.025$ ).



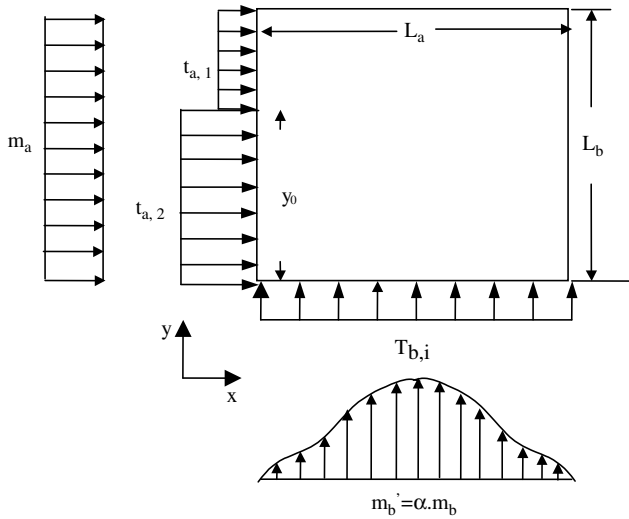


Fig. 10. Schematic representation of crossflow heat exchanger with combined nonuniformity in temperature and flow.

is considered only in hot fluid stream and flow nonuniformity is considered only in cold fluid stream. Combining the two effects, hot and cold fluid mean exit temperature

responses are calculated for different input disturbances in hot fluid, responses have been compared with the corresponding results considering nonuniformity only in temperature, only in flow and that without nonuniformity as shown in Fig. 11.

For step, ramp and exponential excitation, the mean exit temperature of the hot fluid will be the lowest at any instant when no nonuniformity is present either in velocity distribution or in the inlet temperature distribution. The situation will be the reverse for all these excitations when nonuniformities exist both in the temperature and in the flow field. The responses are also similar for the cold fluid mean exit temperature – being highest for combined nonuniformity and lowest for no-nonuniformity case. The mean exit temperatures will have intermediate values for both the fluid streams when the flow or temperature nonuniformities are considered separately. However, the effect of flow nonuniformity is prominent in case of hot stream while the temperature nonuniformity has a greater effect on cold fluid exit temperature.

In case of sinusoidal excitation, the response of hot fluid mean exit temperature is influenced only marginally by any of the nonuniformity or by their combined effect. The effect

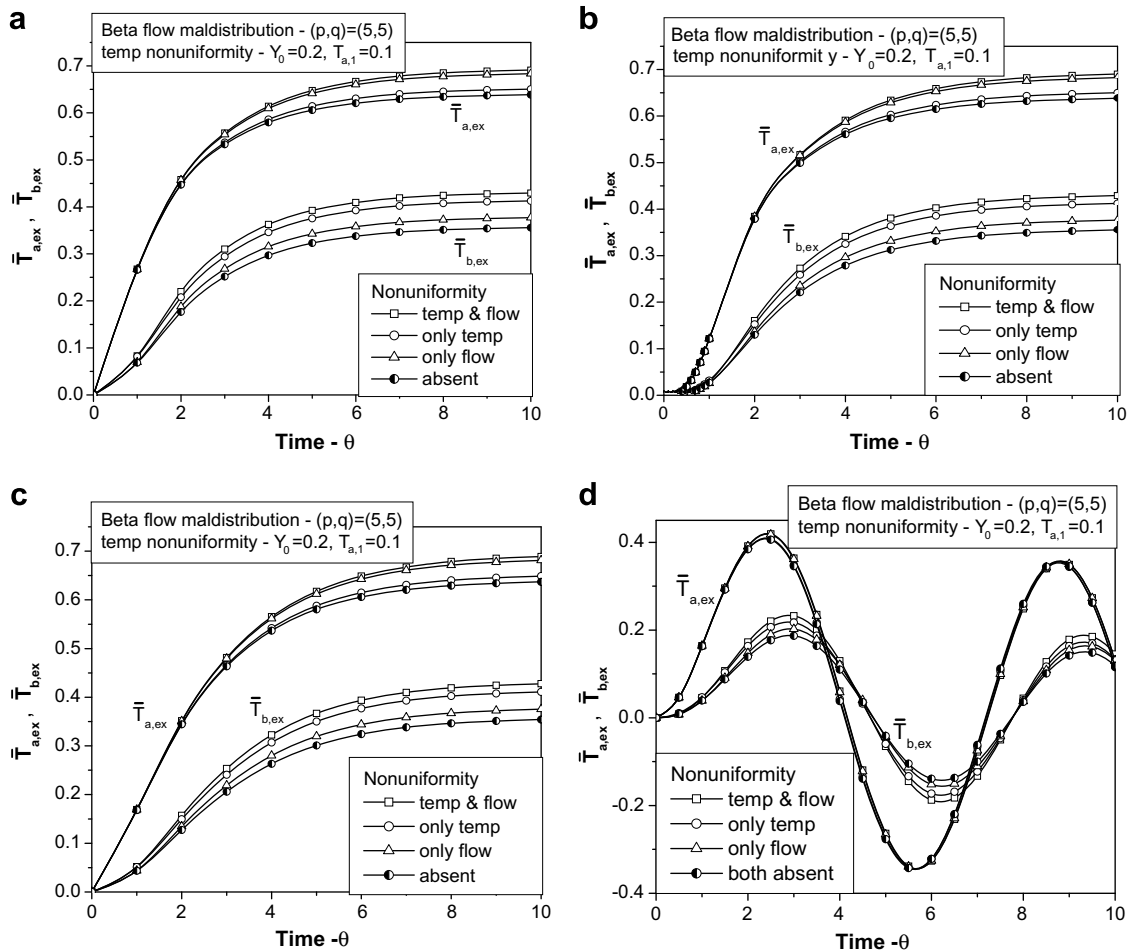


Fig. 11. Combined effect of temperature and flow nonuniformity on hot and cold fluid mean exit temperatures for (a) step, (b) ramp, (c) exponential, and (d) sinusoidal inputs ( $E = R = V = Pe = 1$ ,  $NTU = 2$ ,  $\lambda = 0.025$ ).

on the cold fluid exit temperature is relatively more significant. In general, the amplitude of the cold fluid exit temperature increases due to the presence of nonuniformities, the maximum amplitude observed when nonuniformities are present both in temperature and flow fields.

It may also be noted that the results presented above is dependent on the operating conditions and the parameters selected for specifying the nonuniformities.

## 5. Conclusion

The effects of temperature and flow nonuniformities on the transient response of crossflow heat exchangers have been analysed. Variation in the inlet temperature of the hot fluid is considered in terms of step, ramp, exponential and sinusoidal disturbances and its effect is shown on the mean exit temperature of hot and cold fluids for temperature and flow nonuniformities. It is seen that the performance depends upon the given set of fluid stream temperatures and their relative positions. In most of the cases, the change in the performance of cold fluid is more significant than that of hot fluid. The magnitude of deterioration is found to be dependent on the flow distribution model, i.e. the position of the fluid moving device with respect to the heat exchanger axis. The combined effect of temperature and flow nonuniformity has also been reported, which can give the complete idea of the nature and amount of deterioration in performance of a crossflow heat exchanger in the worst possible situation.

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